Early Structural Reasoning. Gentzen 1932

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Gentzen's first published paper, Über die Existenz unabhängiger Axiomensysteme zu unendlichen Satzsystemen, has recently been stimulating an increasing attention. Many interesting insights into this paper of Gentzen's are for instance provided by Peter Schroeder-Heister in 2002, Curtis Franks in 2010 and 2013, and by Neil Tennant in 2010. (The last paper contains also remarkable generalizations to cases not considered by Gentzen, as for instance to sentences with empty antecedent or succedent, and to consequences of infinite sets of sentences.). One of the reasons of this increasing interest is that, besides witnessing the relevance of Hertz's work for the young Gentzen, it allows us to know that "structural reasoning" predates the formulation of Natural Deduction and Sequent Calculus. By structural reasoning I mean reasoning based on structural rules which affect the way formulas are arranged in consequence relations expressed by sequents, without referring to their internal articulation by means of logical connectives and quantifiers. This means that moving to Sequent Calculus, though strictly linked to difficulties encountered in the attempt to get the Hauptsatz within the framework of Natural Deduction, was anyway a step which extended a perspective already present in Gentzen's work. The talk is organized as follows. The first two sections outline the main features of Gentzen's system: the notation he uses and the characterization provided for the notions of inference, proof, and consequence. It is moreover recalled how he introduced, with respect to this system, the questions of its soundness and completeness. In section 3 we present a proof of the completeness theorem which exploits mainly semantic tools. Gentzen's original argument is retraced in section 5, and some points useful for its theoretical completion are investigated in section 6. In the last section we try in conclusion to place Gentzen's completeness theorem within his general proof-theoretic framework.