Category forcings and generic absoluteness: steps towards a “complete" axiom system for set theory

Matteo Viale

Forcing is the most efficient method to produce independence results in set theory.

Woodin’s work has shown that for problems formulated in second order number theory, forcing becomes a powerful tool to prove theorems and actually in the theory ZFC+ large cardinals gives a complete semantic for second order number theory with respect to first order derivability. We generalize Woodin’s completeness result to a very large fragment of third order number theory with respect to an extension of ZFC+ large cardinals enriched with strong forcing axioms. Forcing axioms are usually formulated as strengthenings of the Baire category theorem and assert that for large classes of compact Haussdorff spaces X the intersection of $\aleph_1$-many open dense sets of X is non empty. Recently we’ve been able to give a formulation of these forcing axioms in the language of categories and to isolate according to this new characterization a forcing axiom (MM^{+++}) for which the mentioned generalization of Woodin’s result to third order number theory holds with respect to the theory ZFC+large cardinals+MM^{+++}.

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