Category forcings and generic absoluteness: steps towards a “complete" axiom system for set theory

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Forcing is the most efficient method to produce independence results in set theory.

Woodin's work has shown that for problems formulated in second order number theory, forcing becomes a powerful tool to prove theorems and actually in the theory ZFC+ large cardinals gives a complete semantic for second order number theory with respect to first order derivability. We generalize Woodin's completeness result to a very large fragment of third order number theory with respect to an extension of ZFC+ large cardinals enriched with strong forcing axioms. Forcing axioms are usually formulated as strengthenings of the Baire category theorem and assert that for large classes of compact Haussdorff spaces X the intersection of $\aleph_1$-many open dense sets of X is non empty. Recently we've been able to give a formulation of these forcing axioms in the language of categories and to isolate according to this new characterization a forcing axiom (MM^{+++}) for which the mentioned generalization of Woodin's result to third order number theory holds with respect to the theory ZFC+large cardinals$+MM^{+++}$.

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