A relational dual tableau procedure to decide some fragments of logic of relations

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Abstract

We consider logics of binary relations which can be exploited as formalisms to represent various theories, in particular some modal and description logics. The relational logics taken into account here are fragments of the relational logic $\mathsf{RL}(\mathbf{1})$ presented in [6]. Formulae of the logic $\mathsf{RL}(\mathbf{1})$ have the form xRy, where x and y are object variables and R is a relational term constructed from relation variables by means of the standard operations for binary relations.

The representation of non-classical logics in $\mathsf{RL}(\mathbf{1})$ relies on the fact that logical formulae may be treated as relations once their Kripke-style semantics is known. In Kripke-style semantics, formulae are interpreted as sets of objects which may be identified with what is called right ideal relations. This, in the case of binary relations, amounts to saying that the relations satisfy $R\mathbin{;}\mathbf{1} = R$, where "$\mathbf{;}$" is the composition of binary relations and $\mathbf{1}$ is the universal relation.

Relational logics appear to be an appropriate, uniform means to represent a great variety of theories (see [6]). Thus any decision procedure for a relational logic is not simply a single decision method for some theory but it is applicable to the theories which can be expressed in the considered relational logic. Relational dual tableau-based decision procedures can be found in [6] for fragments of $\mathsf{RL}(\mathbf{1})$ corresponding to the class of first-order formulae in prenex normal form with universal quantifiers only, in [4, 5] for the relational logic corresponding to the modal logic K, in [1, 2] for fragments of $\mathsf{RL}(\mathbf{1})$ characterized by some restrictions in terms of type $(R\mathbf{;}S)$, and in [3] for a class of relational logics admitting a single relational constant with the properties of reflexivity, transitivity, and heredity.

We present three fragments of $\mathsf{RL}(\mathbf{1})$ and introduce for each of them a decision procedure based on dual tableaux. The logics considered here are characterized by some constraints enforced in the construction of terms of type $R\mathbin{;}S$ whose lead operator is relational composition. The first fragment admits as left hand side R of such terms relational variables only; the second one also admits, inside R, occurrences of union and intersection of relations; the third one allows R to be the universal constant $\mathbf{1}$. These fragments extend the relational logics presented in previous work since they allow the same relational variable to occur on both sides of terms of type $R\mathbin{;}S$. Finally, we show how some modal and description logics can be expressed by these fragments.

References

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