Interval Temporal Logics and Equivalence Relations

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Interval temporal logics provide a general framework for temporal representation and reasoning, where point-based linear temporal logics can be recovered as special cases. In this paper, we study the effects of the addition of one or more equivalence relations to two meaningful fragments of Halpern and Shoham's modal logic of time intervals [7], namely, $A\overline{A}$ and AB, interpreted over the class of finite linear orders. The logic $A\overline{A}$ is as expressive as the two-variable fragment of first-order logic with a linear order $FO^2[<]$ [3], and there are polynomial reductions of the satisfiability problem from one logic to the other; the logic AB allows one to express significant metric properties, such as, for instance, to constrain the minimal or maximal length of an interval [12].

The trade-off between the increase in expressiveness and the complexity blow-up induced by the addition of one or more equivalence relations to a logic has been already highlighted in the literature (see, for instance, the logics for semi-structured data [2], temporal logics [5], and timed automata [15]). Finite satisfiability of the two-variable fragment of first-order logic FO^2 extended with one, two, or more equivalence relations has been systematically explored in [8–10], while the extension of FO^2, interpreted over finite or infinite data words, with an equivalence relation has been investigated by Bojańczyk et al. in [2]. Similar results have been obtained by Demri and Lazic [5], that studied the extension of linear temporal logic over data words with freeze quantifiers, which allow one to store elements at the current word position into a register and then to use them in equality comparisons deeper in the formula, and by Ouaknine and Worrell [15], who showed that both satisfiability and model checking for metric temporal logic over finite timed words are decidable with a non-primitive recursive complexity. The addition of an equivalence relation to an interval temporal logic was first investigated by Montanari and Sala in [13]. They focused their attention on the interval logic $AB\overline{B}$ of Allen's relations *meets*, *begun by*, and *begins* extended with an equivalence relation, interpreted over finite linear orders and $\mathbb{N}$, and they showed that the resulting increase in expressive power makes it possible to establish an original connection between interval temporal logics and extended regular languages of finite and infinite words [1].

In this work, we first prove that the satisfiability problem for the logic of temporal neighborhood $A\overline{A}$ extended with one equivalence relation, denoted by $A\overline{A}\sim$, is decidable (**NEXPTIME**-complete) over finite linear orders. The proof rests on a model contraction technique that generalizes the point elimination technique used to demonstrate the decidability of $A\overline{A}$ in [4]. First, we show that the size of every equivalence class of a minimal model of a formula φ is (exponentially) bounded in the length of φ; then, we give an exponential bound to the number of equivalence classes in a minimal model. The existence of an (exponential) upper bound to the cardinality of a small model for $A\overline{A}\sim$ easily follows. Moreover, the polynomial reductions from $A\overline{A}$ to $FO^2[<]$ and vice versa can be easily extended to $A\overline{A}\sim$ and $FO^2[<,\sim]$, thus proving

decidability and **NEXPTIME**-completeness of $FO^2[<, \sim]$ (it is worth noticing that $FO^2[<, \sim]$ has the same complexity as weaker fragments such as FO^2, $FO^2[\sim]$, and $FO^2[<]$ [6, 9, 14]). Next, we focus our attention on AB. In [13], Montanari and Sala proved that its extension with one equivalence relation $\sim_1$ is decidable, with a non-primitive complexity, over finite linear orders and undecidable over $\mathbb{N}$. Here, we show that the addition of two equivalence relations $\sim_1, \sim_2$ makes it undecidable over finite linear orders as well. The proof consists of a reduction from the halting problem for (Minsky) counter machines, which is known to be undecidable [11], to the satisfiability problem for $AB\sim_1\sim_2$.

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