

WEAK AND LOCAL VERSIONS OF MEASURABILITY

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Local versions of measurability have been around for a long time. Roughly, one splits the notion of μ -completeness into pieces, and asks for a uniform ultrafilter over μ satisfying just some piece of μ -completeness. This includes notions studied by C. C. Chang, K. Prikry, J. H. Silver and others.

Analogous local versions of weak compactness are harder to come by, since weak compactness cannot be defined by using a single ultrafilter. Our technique to deal with the problem is to restrict just to a subset P of all the partitions of μ into $< \mu$ classes and ask for some ultrafilter D over μ such that no partition in P disproves the μ -completeness of D (the μ -completeness of D might be or might be not disproved by some partition outside P). By making P vary in appropriate classes, one gets both measurability and weak compactness, as well as possible intermediate notions of “weak measurability”, which have been recently proved to be distinct in some model by J. A. Schanker.

We systematize the above procedures and combine them to obtain variants of measurability which are at the same time weaker and local. In particular, we present local versions of weak compactness. Of particular interest is the fact that the notions thus obtained admit equivalent formulations through topological, model theoretical, combinatorial and Boolean algebraic conditions.

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