Defining necessity from contingency: a case study in tense logic

Matteo Pascucci
(Università di Verona)

Systems of modal logic are usually based on a language in which there is a primitive notion of necessity or possibility, while the remaining modalities are introduced by means of auxiliary definitions. The reason is that an exhaustive picture of modalities can be easily obtained when an operator for necessity or possibility and truth-functional connectives are available. On the other hand, the task of defining necessity and possibility from different modal notions presents some technical difficulties and is interesting to explore in order to answer the philosophical question whether all modal notions are on the same conceptual level. Here we are especially concerned with languages containing a primitive notion of contingency (Aristotle's two-sided possibility).

In the Sixties, Montgomery and Routley studied a modal language with ∇ ("it is contingent that") as primitive operator and axiomatized systems deductively equivalent to **KT**, **S4** and **S5**. Given that such systems are interpreted on reflexive frames, one can easily appeal to the following definition to introduce an operator of necessity: $\Box\alpha := \alpha \wedge \neg\nabla\alpha$. In the Nineties, Humberstone and Kuhn provided axiomatizations for the minimal system of contingency logic **K**∇; however, as a consequence of a previous result obtained by Cresswell, no definition of $\Box$ in terms of ∇ and truth-functional connectives is available in **K**∇. Despite this fundamental limit, Pizzi and Zolin proposed alternative methods to define a notion of necessity in weak systems of contingency logic. We will focus on Pizzi's idea of a language enriched with a propositional constant and extend some of his results to the context of tense logic, a bimodal logic with two primitive operators that allow past and future reference from a given instant of evaluation. Systems of tense logic are interpreted on temporal frames $\mathfrak{F} = \langle T, <, > \rangle$ and every normal system contains two bridge-axioms ensuring that $<$ and $>$ are mutually inverse in the relevant class of frames: $\forall t, t'(t < t' \equiv t' > t)$. We will provide such axioms in a language whose primitive operators are ∇_p ("it has been contingent that") and ∇_f ("it will be contingent that") and get a system of tense logic where operators for past and future necessity can be defined. Furthermore, this system can be proved to be sound and complete with reference to a certain class of temporal models.