On proofs and countermodels

(Summary)

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The duality of proofs and counterexamples, or more generally, refutations, is ubiquitous in science and finds a precise expression in mathematics, in well-defined fragments formalized logically.

In a logical formalization, things become more structured because of the possibility to reason within formal *analytic calculi* that reduce the proving of theorems to automatic tasks. Usually one can rest upon a completeness theorem that guarantees a perfect duality between proofs and countermodels. So in theory. In practice, we are encountered with obstacles: completeness proofs are often non-effective (non-constructive) and countermodels are artificially built from Henkin sets or Lindenbaum algebras, and thus far away from what we regard as counterexamples. Furthermore, the canonical countermodels provided by traditional completeness proofs may fall out of the intended classes of models.

The question naturally arises as to whether we can find in some sense "concrete" countermodels in the same automated way in which we find proofs. *Refutation calculi* produce refutations rather than proofs and can be used as a basis for building countermodels. These calculi are separate from the direct inferential systems, their rules are not invertible (root-first, the rules give only sufficient conditions of non-validity), and sometimes the decision method through countermodel constructions uses a pre-processing of formulas into a suitable normal form. These calculi often depart from Gentzen's original proof systems, because the sequent calculus **LI** or its contraction-free variant **LJT** have rules that are not invertible; thus, while preserving validity, these latter calculi do not preserve refutability.

We shall outline a method for unifying proof search and countermodel construction that is a synthesis of a generation of calculi with internalized semantics, a Tait-Schütte-Takeuti style completeness proof and a procedure to finitize the countermodel construction. The methodology of generation of complete analytic countermodel-producing calculi has so far been applied in: intermediate logics and their modal companions; intuitionistic multi-modal logics; provability logics; knowability logic; logics with frame properties beyond geometric theories that cover all the Sahlqvist fragment.

Reference: Sara Negri. Proofs and countermodels in non-classical logics. *Logica Universalis*, in press.