Absoluteness via Resurrection

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Generic absoluteness over a theory T containing ZFC is the phenomena by which the truth value of mathematical statements of a certain logical complexity is invariant with respect to appropriate types of forcing which preserve T. This topic has been studied since the introduction of forcing in the late '60, and is motivated by the broad success that the method of forcing reported on consistency results.

These kind of results for a theory T provide a mean to restrict the independence phenomena dating back to Gödel's incompleteness theorems, and can be used to turn the consistency proofs of certain first order statements ϕ into actual derivations (in first order calculus) of ϕ from T.

We expand on Viale and Hamkins work (among others) on generic absoluteness and resurrection axioms and we introduce the iterated resurrection axioms $\mathrm{RA}_\alpha(\Gamma)$ as α ranges among the ordinals and Γ varies among various classes of forcing notions. Our main results (obtained jointly with Viale) are the following:

Theorem. *If $\mathrm{RA}_\omega(\Gamma)$ holds and $\mathbb{B} \in \Gamma$ forces $\mathrm{RA}_\omega(\Gamma)$, then $H_{\mathfrak{c}}^V \prec H_{\mathfrak{c}}^{V^{\mathbb{B}}}$ (where $\mathfrak{c} = 2^{\aleph_0}$ is the continuum as computed in the corresponding models).*

Hence a statement $\phi^{H_{\mathfrak{c}}}$ regarding the structure $H_{\mathfrak{c}}$ is first order derivable in the theory $T = \mathrm{ZFC} + \mathrm{RA}_\omega(\Gamma)$ whenever T proves its consistency together with T by means of a forcing in Γ.

Theorem. $\mathrm{RA}_\alpha(\Gamma)$ *is consistent relative to the existence of a Mahlo cardinal for the following classes of posets: all, ccc, axiom-A, proper, semiproper.*

We remark that the existence of a Mahlo cardinal is very low in the large cardinal hierarchy.

Theorem. $\mathrm{RA}_\alpha(\Gamma)$ *for Γ the class of stationary set preserving posets is consistent relative to the existence of a stationary limit of supercompact cardinals.*

In the talk we shall motivate the foundational role played by generic absoluteness results, sketch a proof of some of our results, and compare our results to the current literature in the field.