Physical properties as modal operators in the intuitionistic approach to quantum mechanics

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In classical physics, every system can be described by specifying its actual properties. Mathematically, this happens by representing the state of the system by a point (p, q) in the corresponding phase space Γ and, its properties by subsets of Γ. Consequently, the propositional structure associated with the properties of a classical system follows the rules of classical logic. In the orthodox formulation of quantum mechanics, a pure state of a system is represented by a ray in a Hilbert space $\mathcal{H}$ and its physical properties by closed subspaces of $\mathcal{H}$ which, with adequate definitions of meet and join operations, give rise to an orthomodular lattice. This lattice, denoted by $\mathcal{L}(\mathcal{H})$, is called the Hilbert lattice associated to $\mathcal{H}$ and motivates the standard quantum logic introduced in the thirties by Birkhoff and von Neumann [1].

In the last years, several approaches using topos theory, have been used to search for an adequate and rigorous language for quantum systems [2, 3, 4, 5]. In these approaches, the quantum analogue of classical phase space is captured by the notion of frame i.e. an intuitionistic structure.

In this framework, we provide a representation of physical properties as modal operators in a Heyting structure. This representation allows us to analyze the classical and quantum aspects of properties in terms of logical consequences.

References

[1] G. Birkhoff and J. von Neumann, "The logic of quantum mechanics", Ann. Math. **37** (1936) 823843 .

[2] G. Domenech and H. Freytes, "Contextual logic for quantum systems", J. Math. Phys. **46** (2005) 012102.

[3] A. Döring and C.J. Isham, "What is a thing?: topos theory in the foundations of physics" in *New Structures for Physics*, Lecture Notes for Physics **813**, B. Coecke (Editor), Springer, Berlin-Heidelberg, 2010, 753-937.

[4] C. Heunen, N. Landsman and B. Spitters, "A topos for algebraic quantum theory", Comm. Math. Phys. **291** (2009) 63-110.

[5] C. Isham, "Topos methods in the foundations of physics", in *Deep beauty: understanding the quantum world through mathematical innovation*, H. Halvorson (Editor), Cambridge University Press, 2010, 187-206.