A geometrical representation of the basic laws of Categorial Grammar

V. Michele Abrusci and Claudia Casadio

[1]Dept. of Philosophy, University Roma Tre, Rome, Italy - abrusci@uniroma3.it
[2]Dept. of Philosophy, Chieti University, IT - casadio@unich.it

Keywords Categorial grammar, Lambek calculus, cyclic linear logic, proof-net

Abstract

We propose a geometrical representation of the laws that are at the basis of categorial grammar, and particularly of the Lambek calculus [5], in the framework of *Cyclic Multiplicative Linear Logic*, a purely non-commutative fragment of linear logic [1]. In [2] it is argued that the principles of categorial grammar [3, 4] and of the Lambek calculus, known in the literature as *Residuation* laws, can be characterized in the framework of Cyclic Multiplicative Linear Logic. In a sequent calculus style the Residuation laws state the following equivalences between sequents of a suitable formal language: for every formula A, B, C

(RES) $A \cdot B \vdash C$ iff $B \vdash A \backslash C$ iff $A \vdash C/B$, where

- $\cdot$ is the *residuated* connective, the *conjuntion*, denoted by $\otimes$ in linear logic;
- $\backslash$ is the *left residual* connective, the left implication, denoted by $\multimap$ in linear logic;
- $/$ is the *right residual* connective, the right implication, denoted by $\circ\!-$ in linear logic.

The result obtained in [2] is to show that the sequents occurring in the formulation of the residuation laws are exactly all the possible ways to describe the conclusions of the same proof net with three conclusions in Cyclic Multiplicative Linear Logic. More generally, the residuation laws represent the equivalence between the 10 sequents which are all the possible intuitionistic descriptions of the conclusions of the same Cyclic Multiplicative Proof Net with three conclusions.

In the present work we analyze a larger family of categorial grammar laws and rules [cf. 3], specifically:

- (a) *Monotonicity* laws: $(A \vdash B) \vdash (A \otimes C \vdash B \otimes C)$ and $(A \vdash B) \vdash (C \otimes A \vdash C \otimes B)$
- (b) *Functional application* rules: $C/A, A \vdash C$ and $A, A\backslash C \vdash C$
- (c) *Type raising* rules: $A \vdash (C/A)\backslash C$ and $A \vdash C/(A\backslash C)$
- (d) *Functional composition* rules: $(A/B) \otimes (B/C) \vdash (A/C)$ and $(A\backslash B) \otimes (B\backslash C) \vdash (A\backslash C)$
- (e) *Geach* rules: $A/B \vdash (A/C)/(B/C)$ and $A\backslash B \vdash (C\backslash A)\backslash(C\backslash B)$

The geometrical representation of *Monotonicity* laws (a) is obtained by taking two objects: an arbitrary proof net with two conclusions and a proof net only consisting of an axiom link. In the particular case in which we take two proof nets each consisting of a single axiom link, one obtains a geometrical representation of the rules of *Functional application* (b) and *Type raising* (c). As a further generalization, the geometrical representations of *Functional composition* rules (d) and *Geach* rules (e) are obtained by considering three proof nets each consisting of a single axiom link.

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