

p -adic numbers, exponential ring and decidability - Abstract

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Unlike the real and complex fields, a very little is known about the p -adics in the context of exponential fields. An issue when considering the field of p -adics is that there is no natural structure of exponential field. The map $\exp(x)$ determined by the power series $\sum x^n/n!$ is convergent iff $v(x) > 1/(p-1)$. Yet, one can define using this map a structure of exponential ring on the ring of p -adic integers \mathbb{Z}_p . Let $E_p(x) := \exp(px)$ (if $p \neq 2$, we set $E_2(x) := \exp(4x)$). Then, $E_p(x)$ is well-defined for all $x \in \mathbb{Z}_p$ and $(\mathbb{Z}_p, +, \cdot, 0, 1, E_p)$ is an exponential ring (i.e. a ring equipped with a morphism of groups between its additive and multiplicative groups). In this talk, I will present some results on the model theory of this structure. The main result known about this structure is that its theory is decidable if a p -adic version of Schanuel's conjecture is true. As time allows, I will expose some aspects related the proof of this theorem.