Space Complexity in Algebraic Proof Systems

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The study of space measure in Proof Complexity has a gained in the last years more and more importance: (1) it is clearly of theoretical importance in the study of complexity of proofs; (2) it is relevant to SAT solving, since it might provide theoretical explanations of efficiency or inefficiency of specific Theorem Provers or SAT-solvers; (3) it is connected with important characterizations studied in Finite Model Theory, thus providing a solid link between the two research fields.

In this work where we devise a new general combinatorial framework for proving space lower bounds in algebraic proof systems like Polynomial Calculus (PC) and Polynomial Calculus with Resolution (PCR). Our method can be view as a Spoiler-Duplicator game, which is capturing boolean reasoning on polynomials. Hence, for the first time, we move the problem of studying the space complexity for algebraic proof systems in the range of 2-players games, as is the case for Resolution. This can be seen as a first step towards a precise characterization of the space for algebraic systems in terms of combinatorial games, like Ehrenfeucht-Fraisse , which are used in Finite Model Theory.

A simple application of our method allows us to obtain all the currently known space lower bounds for PCR, like the Pigeonhole Principle. More importantly, using our approach in its full potentiality, we answer to the open problem of proving space lower bounds in Polynomial Calculus and Polynomials Calculus with Resolution for the polynomial encoding of randomly chosen k-CNF formulas. Our method also applies to the the well studied Graph Pigeonhole Principle which is a Pigeonhole principle over a constant (left) degree bipartite expander graph.

The work arises several open problems might be solved generalizing our approach.