An alternative proof-theoretical approach to standard conditional logics

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Conditional logics, which have a long and venerable history [3, 1, 2], have been introduced to capture counterfactual sentences, i.e. conditionals of the form "if A were the case, then B would be the case", where A is false. If we interpret counterfactuals as material implications, we have that all counterfactuals are trivially true, and this is an unpleasant conclusion. By means of conditional logics, on the other hand, we can give a different and meaningful interpretation of counterfactual sentences.

There are several different systems of conditional logics. Amongst them we focus on the system CK and its standard extensions, namely CK + {ID, MP, CEM}. These systems have a simple and useful semantics. One just needs to consider a set of possible worlds W, and a selection function f; for each world i and each formula A, f selects the set of worlds of W which are *closer* to i given the information A. Thus a counterfactual sentence $A > B$ is true at a world i if, and only if, B is true at all those worlds that are closer to i given the information A.

In this talk we aim at presenting a method for generating sequent calculi and natural deduction calculi for the system CK and its extensions. The method is based on and fully exploits the simple semantics interpretation of such systems. Sequent calculi and natural deduction calculi are proved to be equivalent; moreover, as for the natural deduction calculi, we prove that the derivations normalize; while, as for the sequent calculi, we prove they are contraction-free, weakening-free and cut-free, and that their logical rules are all invertible.

References

[1] Lewis, D. *Counterfactuals*. Basil Blackwell, 1973.

[2] Nute, D. *Topics in Conditional Logic*. Reidel, Dordrecht, 1980.

[3] Stalnaker, R. A theory of conditionals, American Philosophical Quarterly, Monograph Series no.2, Blackwell, Oxford, 98–112, 1968.