

The Hurewicz dichotomy for generalized Baire spaces

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Hurewicz showed that every analytic subset of the Baire space ${}^\omega\omega$ is either covered by a countable union of compact sets, or else it contains a closed set homeomorphic to ${}^\omega\omega$. We show that it is consistent that the analogous statement for the generalized Baire space ${}^\kappa\kappa$, called the Hurewicz dichotomy for κ , holds for any given uncountable cardinal κ such that $\kappa^{<\kappa} = \kappa$. Moreover, it is also consistent that the Hurewicz dichotomy holds simultaneously at all uncountable regular cardinals κ , as well as that it fails simultaneously at all uncountable regular κ . We further show that the combinations of the Hurewicz dichotomy with the perfect set property and with its negation are both consistent. Finally, we present two applications of these results concerning a topological characterization of κ -Canjar filters and a combinatorial regularity property called κ -Miller measurability.