

Diophantine Algebraic Geometry for MV-Algebras

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MV algebras are the algebraic counterparts of Lukasiewicz many valued logic, playing a role analogous to Boolean algebras in classical, two valued logic. In this paper we try to apply universal algebraic geometry (in the sense of Plotkin, Sela and others) to MV algebras, obtaining a Nullstellensatz result and a categorical duality between “MV algebraic sets” given by zeros of MV polynomials, and their “coordinate MV algebras”. Next, we focus on the structure of MV polynomials and MV polynomial functions on a given MV algebra, obtaining also a seemingly new notion of “polynomially complete” MV algebra (intuitively an MV algebra where MV polynomials and MV polynomial functions coincide) which we begin to study. Finally we relate algebraic and geometric objects with theories and models taken in Lukasiewicz many valued logic with constants. Our algebraic geometry is Diophantine in the sense that the MV algebra where we take coefficients of polynomials is the same MV algebra where we evaluate polynomials.