Simmetry in sequent calculus from quantum computation

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The problem of modelling quantum mechanics by logic has been acquiring more and more relevance since the birth of the idea of quantum computer, that is of exploiting quantum processes for computation and communication. In the quantum framework alternative approaches are needed, with respect to the traditional view of computation, concerning the logical foundations: As is becoming more and more apparent, quantum entanglement [3], that is the key for quantum speed up, subverts the usual, input-output oriented, that is functional, view of algorithms, in favour of a different unoriented view of the information links. Our model aims, eventually, to import such a view into logic, and to see if the oriented and the unoriented environments are compatible in some way or not. We consider the definition of connectives in sequent calculus, via equations, in the approach of basic logic [4]. This permits to describe pure quantum states by quantifiers [1]. First order variables are so adopted in order to capture the intrinsic kind of randomness of quantum mechanics. We interpret our first order domains as "infinite singletons". We see that infinite singletons cut out a symmetric kernel from sequent calculus, where negation is meaningless and the orientation of the sequent itself is irrelevant [2]. It is a really intriguing open problem to see how the implication could then arise, when infinite singletons disappear and the intrinsic randomness of quantum mechanics is dropped in favour of determinism. A parallel aspect of the problem is its linearity, namely, how the structural rules of Gentzen could be considered as originated in the symmetric kernel.

References

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