Nonstandard Techniques in Combinatorial Number Theory

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In the last few years many results in combinatorial number theory have been proved by means of nonstandard techniques. The aim of our talk is to present some of these techniques by means of some selected examples of applications. A particular attention will be given to the study of the following kind of problems: given a diophantine equation $P(x_1, ..., x_n) = 0$ we say that this equation is partition regular on N if, for every finite coloration of N, there exist monochromatic natural numbers $a_1, ..., a_n$ such that $P(a_1, ..., a_n) = 0$. While everything is known when $P(x_1, ..., x_n)$ is a linear polynomial, the situation is far more complicated in the nonlinear case. For example, even the simple problem regarding the partition regularity of the equation $x + y = zw$ took a few years to be solved. Starting from a nonstandard proof of the partition regularity of this equation, we will generalize this proof to include a wider class of equations $P(x_1, ..., x_n) = 0$. Finally, we will also present a few general conditions that ensure a negative answer to the question regarding the partition regularity of a given equation $P(x_1, ..., x_n) = 0$.